Special Relativity: an astonishing demo of Lorentz equations using the principle of relativity only.[1],[2]

Introduction

In his 1905 paper, for deriving Lorentz transformations, Einstein set 2 postulates.

1- Principle of relativity:

All inertial systems (Galilean reference frames) are equivalent. It is a total departure from the classical conception which postulated an absolute space, (ether) which was the reference.

In Einstein's conception, as there is no absolute Galilean frame of reference, we can no longer speak of the "absolute" velocity of a Galilean reference frame. We can only define a relative velocity between them.

2- The velocity of light is the same in all Galilean inertial systems.

3- First consequences

These considerations, which seem “a priori” almost "trivial", will have serious consequences. Space and time, although considered as immediate data of our consciousness, are no longer the fundamental entities of a physical nature.

As Minkowski said, space and time are no more than shadows of a new fundamental entity: the spacetime.

We demonstrate that the principle of relativity, alone, allows to get the full demonstration of Lorentz equation. Therefore, the second principle is accessory as it just specifies the value of a constant according experimental data.

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[1] *Reference J.M Levy-Leblond. https://dspace.ist.utl.pt/bitstream/2295/52597/1/Levy-Leblond_(76).pdf.) and some others, see for instance (http://www.tree-man9621.com/PDF%20LEVY-LEBLOND%20DID%20NOT%20CREATE%20LORENTZ%20TRANSFORMATIONS.pdf)*

[2] The demonstration is made for a one space dimension. It can be extended to full space.

Annex

The astonishing heuristic power of the relativity first postulate

Starting with using the general constraints of the symmetry of the problem

Let us describe the Lorentz relation between 2 axis Ox and O'x' sliding one on the other with a constant velocity V

--------------------------------O'---------------------------->x'

OO'=V

--------------------------------O----------------------------- >x

To get a perfect symmetry between the two frames let us inverse the direction of the O'x ' axis
x'<------------------------------O'----------------------------------

---------------------------------O---------------------------------> x

The homogeneity of the problem implies a linear transformation and if we select t = t' = 0 at O and at O' at crossing time, the transformations (x, t) → (x', t') and (x', t')→ (x, t), listed below, will include 8 constants labelled from A to D'

(4) $\mathbf{x' = Ax + Bt}$ $\mathbf{t' = Cx + Dt}$

$\mathbf{x = A'x' + B't'}$ $\mathbf{t = C'x' + D't'}$

The relativity principle and the symmetry imply that:

(5) $\mathbf{A = A'}$ $\mathbf{B = B'}$ $\mathbf{C = C'}$ $\mathbf{D = D'}$

Moreover, in O', x' = 0 and x = Vt, therefore x' = Ax + Bt implies AV + B = 0, as well as x = Ax' + Bt' et t = Cx' + Dt' imply B = DV, thus D = -A.

Finally, for consistency

(6) x = Ax' + Bt' = A(Ax + Bt) + B(Cx + Dt) = (D' + CDV)X t = Cx' + Dt' = C(Ax + Bt) + D(Cx + Dt) ='(D' + CDV) t

Therefore, $D^2 + CDV = 1$, i.e: $C = \frac{1-D^2}{DV}$

The transformation (x, t) ⟶ (x', t') becomes, then :

(7) $\mathbf{x' = -Dx + DVt}$ $\boldsymbol{t' = \frac{1-D^2}{DV}x + Dt}$

The sole parameter left unknown, D, is a function of the velocity V. This can be find by comparing the relations involving several frames of different velocities.

Using the group structure of the Lorentz transformations

Going back to O'x' , now let us consider 3 axis Ox, O'x' et O"x" of same orientation

---------------------------- O''--------------------------------------->x''

OO''=V''t, O'O =V't' '

--------------------------------O'--------------------------------------->x'

OO' = Vt

---------------------------------O--------------------------------------->x

Relation (7) becomes, with the opposite sign, for x'

(8) $$x' = D(x - Vt) \qquad t' = \frac{1-D^2}{DV}x + Dt$$

And the same for D' for r V' and D" for V "(this new D' is independent of that of(4)-(5), which is no longer used after (5)):

(9) $$x'' = D'(x' - V't'), \quad t'' = [(l - D'^2)/(D'V')]x' + D't'$$

(10) $$x'' = D''(x - V''t) \quad t'' = [(l - D''^2)/(D''V'')]x + D''t$$

Let us eliminate x' and t' in (9) by using (8), we get another expression of (10)
Identifying (10) and (11) give four relations as follow

(12) $$D'' = DD' + [D'V'(D^2-1)/(DV)]$$

(13) $$D''V'' = DD'(V + V')$$

(14) $$(1 - D''^2)/D''V'' = [(D - DD'^2)/(D'V')] + [(D' - D^2D')/(DV)]$$

(15) $$D'' = DD' + [DV(D'^2 - 1)/(D'V')]$$

Thus, with (12) et (15*):*

16) $$D'' - DD' = D'V'(D^2 - 1)/(DV) = DV(D'^2 - 1)/(D'V')$$

A constant will emerge from this relation

The last equation allows to defines the parameter K such as:

(17) $$K = D^2V^2/(D^2-1) = D'^2V'^2/(D'^2 - 1)$$

This parameter K has the same value for two different arbitrary velocities with their associated parameter D),

Therefore, K is a constant, not depending of the relative velocity between the Galilean frames. As when V = 0 we get x = x' and t = t', therefore D = 1 in (8), we must select the positive of (17)

(18) $$\boldsymbol{D} = \frac{1}{\sqrt{1-\frac{\boldsymbol{V}^2}{\boldsymbol{K}}}}$$

With (8) we get the transformation (x, t)→(x', t) and Poincaré extend it easily to the general transformation (x, y, z, t) → (x', y', z', t').

(19) $$x' = \frac{x-Vt}{\sqrt{1-\frac{V^2}{K}}}, y' = y,\ z' = z \qquad t' = \frac{\boldsymbol{t}-\frac{\boldsymbol{Vx}}{\boldsymbol{k}}}{\sqrt{1-\frac{\boldsymbol{V}^2}{\boldsymbol{K}}}}$$

Which value for the constant K?

It would provide (1) if it is infinite corresponding to the Galilean mechanics and a Lorentz transformation if $K = c^2$. Obviously these 2 transformations are not very different when V <<c.

The constant K cannot be negative (this would allow backward time flow) and its square root appears as an upper limit for the relative velocity between 2 Galilean frames.

This is confirmed by the square root $\mathbf{(1-V^2/K)^{1/2}}$ and by a law of combination of velocities deducted from (12) and (13):

(19) V’’ = (V+V’) / [1 + (VV’/ K)] by setting $\mathbf{K = k^2}$

(19) (k-V’’)/(k+V’’) = [(k-V)/(k+V)].[(k-V’) / (k+v’)]

Therefore |V | and |V’| < k imply |V’’ | < k.

Thus│V│ and│V'│ < k implies│V"│ < k.

Poincaré and Lorentz selected $K = c^2$, in accordance with the constancy the light velocity and with the conservation of the Maxwell’s equations in Galilean frames.

End of' annex