Cosmology. Hubble constant H_0:A way for reconciling the measurements made at z >> 1 with those at z <1? Ed.4, 10/26/22

Introduction

Currently, this problem of incompatibility, between the values of H_0 at $z >> 1$ (that using the CMB (Cosmological Microwave Background), giving a value of H_0 of about 67.27 ± 0.6 km / s / Mpc, and those of many methods based on observations at $z <1$ (SN1A, Cepheids, etc.) giving a value of H_0 of about 73.52 ± 1.62 km / s / Mpc, seems to question the standard cosmological model.

At first, one questioned the accuracy of the observations, but these values deviate by more than 4 σ from their probabilistic diagram so, as so many measurements have been made and verified, this hypothesis is less and less credible.

Today, we are in expectation, fearing for some and hoping for others, a questioning of the standard model and therefore the theory of general relativity which supports modern cosmology. The history of science teaches us that no theory is definitive, but currently, there is not any so effective alternative theories to replace it.

What a theory physically predicts depends on the parameters associated with it and the assumptions made. Let us recall that the assumptions of homogeneity and isotropy of the universe are drastic approximations which, even on a large scale (matter is gathered in filamentous structures with gigantic voids), are far from being really satisfied.

Inhomogeneous and anisotropic models of universes are studied, without too much success so far. We also know that 95% of what generates the dynamics of the universe (dark matter and dark energy) are of unknown nature, despite important research.

On the other hand, the inflation paradigm, which solves a few problems, still looks as an ad hoc theory waiting for some experimental evidences.

. However, before proclaiming the fall of the cosmological standard model, it is worth considering whether it is not its parameters that are wrong.

Here, after a presentation of the problem, on an arbitrary example, we show how the value of H_0 depends on the cosmological parameters. In this document, we assume that the fundamental notions of cosmology are known (Robertson-Walker metric, Friedman-Lemaître equation, density parameters Ω_i, cosmological redshift z, luminosity distance d_L, angular distance d_A, the equation of state cosmological fluids, etc.

If not, and if necessary, see, for instance: http://www.astro.ucla.edu/~wright/cosmolog.htm

Brief survey of measurement methods used to determine H_0.

An essential difference in these methods is the value of the redshift of the observed and measured phenomenon.

Method like SN1A, considered as a standard candle (light source whose intrinsic luminosity is known), for example, where z <1, uses the distance of luminosity d_L deduced from an observable which is the measure of the power of light of the object considered (by a device associated with the telescope measuring the energy of the photon flow) that we will combine with the redshift z (measured by a spectrometer on the telescope), which is another observable of the object considered.

This makes it possible to plot $z(d_L)$ curves for different values of z that we will compare with those predicted by the different models and to eliminate some of them and keep others as possible (best matching method).

Which models are compatible with the experimental data?

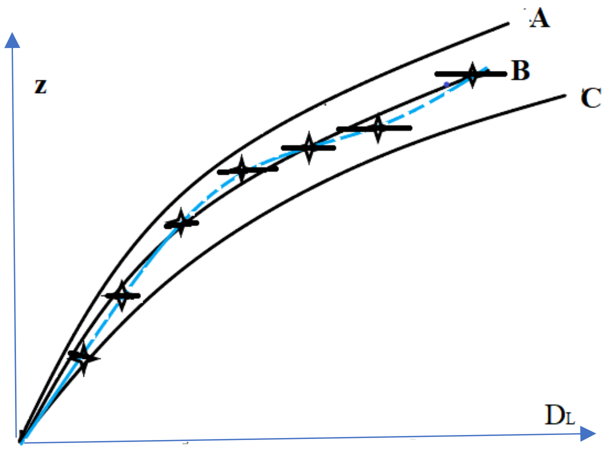


Figure 1: Selection method by best matching to experimental data.

On the diagram above, a set of measurement points for the observations of the redshift z has been represented by star, as a function of the luminosity distance d_L. A dashed curve, connecting them, interpolates the experimental law $z(d_L)$. Each point must be associated with an error bar linked to the inaccuracy of the measurement. We have drawn 3 curves A, B, C corresponding to 3 different cosmological models. We see that for example the curve B is the most compatible with the experimental data. On the other hand, curves A and C are to be excluded.

It is this best fit between observations that will determine which models are compatible with the observations and exclude those that are too far from them, taking into account measurement inaccuracies. Note that this diagram is approximate and only claims to illustrate the phenomenology described.

Luminosity distance

In Euclidean space, the light power F (energy of light received per unit area and per unit of time) from an isotropic source (the Sun for example) of total luminosity L is equal to:

$$F = \frac{L}{4\pi r^2} \qquad (1)$$

where r is the distance between the receiver and the source.

This is simply explained by the fact that the flow is distributed over the surface of a sphere of radius r whose area is $4\pi r^2$. In a non-Euclidean space, other phenomena must be taken into account.

In an expanding space, as described by the Robertson -Walker metric, the emitted photons will be redshifted by the expansion therefore their energy will be divided by

$$1 + z = \frac{a(t_0)}{a(t_e)} \qquad (2)$$

Where z is the observed redshift of the source and $a(t)$ are the scale factors at the emission time of the photon at t_e and at the reception of the photon t_0, (now). A second phenomenon also occurs, the time interval between photons increases all along the geodesic, due to the expansion of space, by a factor also equal to (1 + z), this reduces the received light power.

We will define the luminosity distance d_L, by the relation:

$$F = \frac{L}{4\pi r^2 . a(t_0)^2 (1+z)^2} = \frac{L}{4\pi d_L^2} \qquad (3)$$

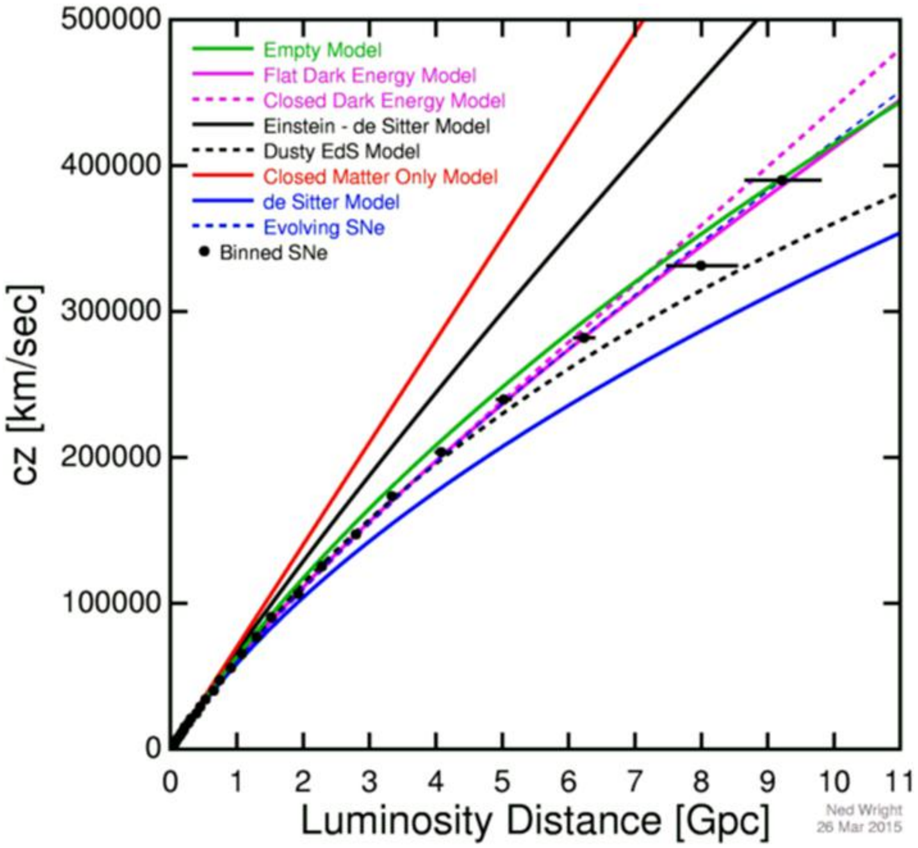
Which gives :

$$d_L = a(t_0) r (1 + z) \qquad (4)$$

Note that in equation (3), we do not know r which is a coordinate, but d_L is defined by (3) since L is a standard candle of known absolute luminosity and F is measured by the observer.

Similarly, z is an observable (redshift of the object, measured by a spectrometer).

Let us recall that it is by the diagrams representing the function $z(d_L)$ that the acceleration of the expansion was detected (1998) and that it allowed, as such, to reintroduce the cosmological constant in the standard model, by using the method of the best fit to the experimental data as described in Figure 1,



In addition to discriminating between different cosmological models, the luminosity distance, a simplified expression of which is given by equation (5) below, shows that this luminosity distance depends on the Hubble constant H_0.

It can therefore be used to estimate the value of the Hubble constant. Its general expression is quite tricky, it is simplified if the spatial curvature of the universe is (about) zero, (**$\Omega_k = 0$**).

Let's make this assumption, consistent with the current assumption on spatial curvature.

This luminosity distance d_L from the object located at a spectral shift z * is given by:

$$d_L = \frac{1+z}{H_0}\int_0^{z*}\frac{dz}{\sqrt{\Omega_{rad}(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}} \qquad eq\ 5$$

Note that the expansion parameter is not *t* but *z*, an observable. This required a transformation which is described in the appendix. This equation includes H_0, the value of *H* for $z = 0$, which is a constant in equation 5 (therefore can be outside of the integral). *H* is linked to H_0 by the formula:

$$H = H_0\sqrt{(\Omega_{rad}(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda)} \qquad eq.6$$

Planck and WMAP calculated the value of H_0 by using the Fourier analysis in spherical waves of the inhomogeneities of the CMB which provides the position of the first peak which is an angle which is the value of the angular view size of the sonic horizon. The size of this horizon, in turn, will be calculated by the distance traveled by sonic waves up to the time of baryons/photon decoupling, this depending of the sonic velocity of these waves in the plasma. These two data will provide the angular distance d_A, whose general definition is the distance associated to the viewed angle of an object of known size.

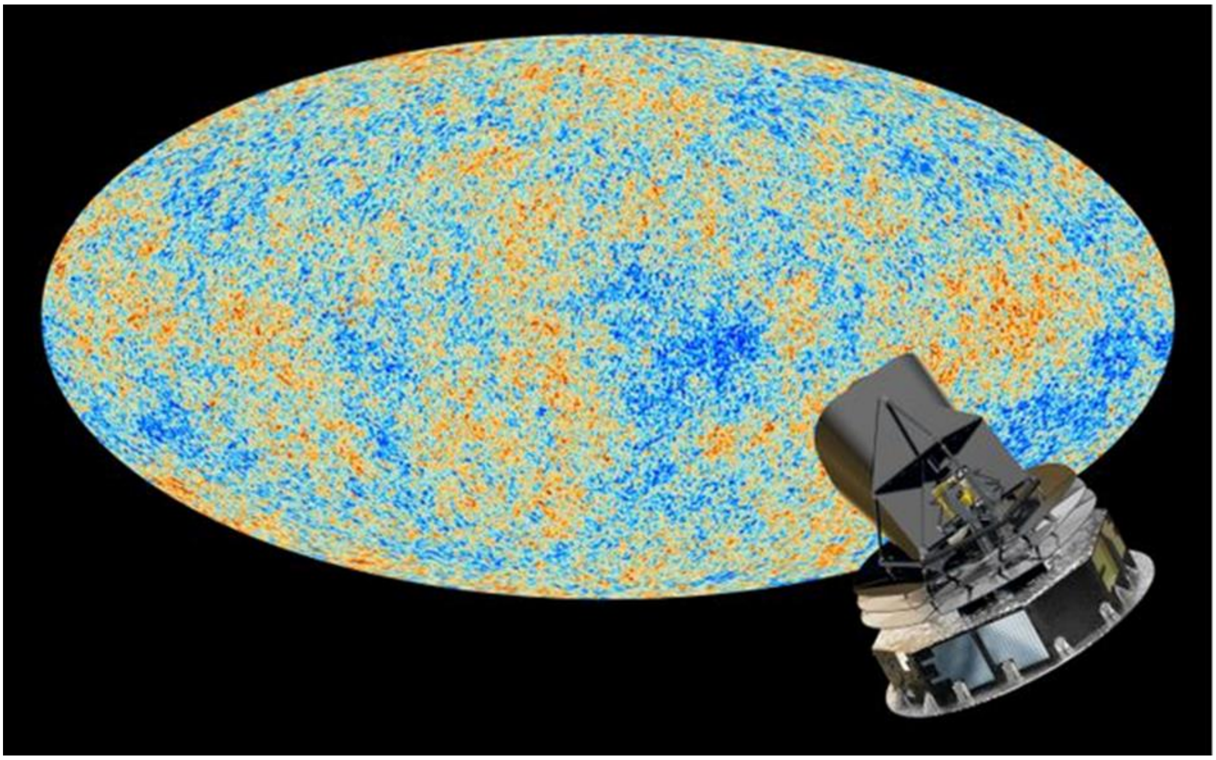


Figure 2: Representation of the temperature of the CMB in "false colors".

The image corresponds to the complete celestial sphere, the Planck Satellite being at the Lagrange L_2 point, 1.5 million km from Earth. All the information is contained in this image.

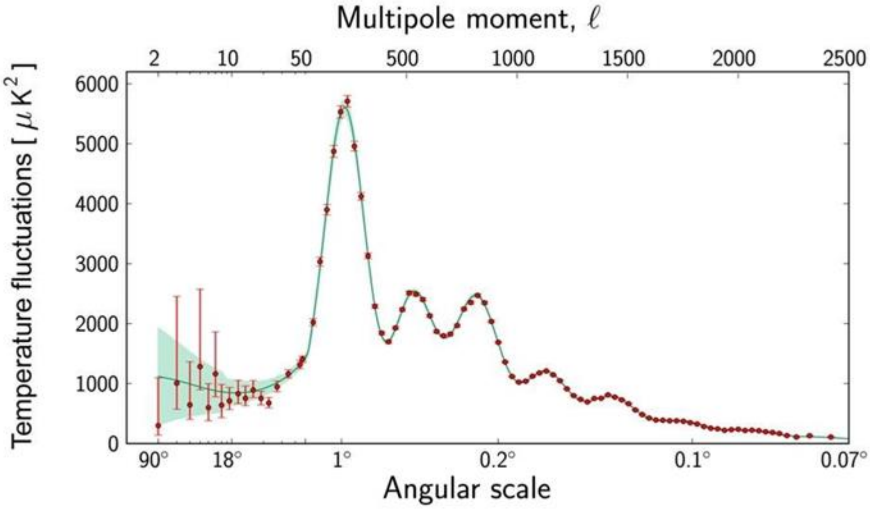


Figure 3: Result of the 2D Fourier transform of the RFC.

This transform extracts the proportion of patterns associated with each size identified by the value of the multipole or the associated angle.
The plasma being an elastic medium, the inhomogeneities generate acoustic waves, therefore the peaks which correspond to the most intense modes (mainly the fundamental mode, corroborated by its harmonics). This defines the size of the sonic horizon in the plasma.
There is a simple relationship between d_L and d_A:

$$d_A = \frac{d_L}{(1+z)^2} = \frac{1}{(1+z)H_0}\int_0^{z*}\frac{dz}{\sqrt{\Omega_{rad}(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}} \quad eq\ 7$$

d_A, therefore, also depends on H_0. Let us suppose, for example, that the cosmological constant, instead of being constant, depends on z (and therefore on t). Under these conditions it is necessary to introduce a factor $f(z)$ associated with the cosmological constant λ, which will have a significant effect for $z >> 1$, while keeping the value for $z = 0$. This will introduce a difference of phenomenology.

$$d_A = \frac{1}{(1+z)H_0}\int_0^{z*}\frac{dz}{\sqrt{\Omega_{rad}(1+z)^4 + \Omega_m(1+z)^3 + f(z)\Omega_\Lambda}} \quad eq\ 8$$

The law that we are going to propose is an example and does not claim any physical character, this will aim to show that the effect can be significant.
In the equation (8), if the integral increases, since the experimental value of d_A is a given experimental data [1], the factor H_0, must increase for keeping d_A constant.
The exact impact of this correction is likely quite complex, but an empirical example modifying the vacuum equation of state of the cosmological constant is given as an example to illustrate the mechanism.

Numerical example

We are interested in the case z >> 1 (method using the results of Planck which are more recent than those of WMAP).

$\Omega_m = 0.306$

$\Omega_{rad} = 0.00009236$

$\Omega_\Lambda = 0.694$

[1] In fact, dA is deduced from the analysis of CMB and cosmological parameters. The size of the "sonic" horizon is deduced from the time until decoupling and from the velocity of sound (velocity of propagation of inhomogeneities in a photon-baryon plasma) and the angle of sight is given by the Fourier transform of the CMB (position of the first peak).

Let's write the part of the equation that describes the influence of H_0 with these conventions and values:

a) Case where the cosmological constant does not vary

$$\frac{1}{H_0}\int_0^{1089} \frac{dz}{\sqrt{0{,}00009236(1+z)^4 \; + 0{,}306(1+z)^3 \; + 0{,}694}} \qquad eq\ 9$$

b) Case where the cosmological is no longer constant and is a function of z while keeping its value for $z = 0$.

Let us say, for example

$$f(z) \; = \; 10^{-z^{10^{-3}}}$$

which satisfies these constraints.

$$\frac{1}{H_0}\int_0^{1089} \frac{dz}{\sqrt{0{,}00009236(1+z)^4 \; + 0{,}306(1+z)^3 \; + (0{,}694)10^{-[z^{(10^{-3})}]}}} \quad eq\ 10$$

a) In this case, the value of the integral given by mathematica's "NIntegrate" (numerical integration) function is : 3.15393

b) In this case, the result given by mathematica by the function NIntegrate is : 3.43586

Deduced value of H_0

Let us Recall the equation that governs these parameters

$$d_A = \frac{c}{(1+z)H_0}\int_0^{z*} \frac{dz}{\sqrt{\Omega_{rad}(1+z)^4 \; + \Omega_m(1+z)^3 + f(z).\Omega_\Lambda}}$$

We see that if we know d_A and the value of the integral, since we know the redshift z and as c is the velocity of light, we can deduce H_0. If the value of the integral increases the value of H_0 (which is the denominator) must also increase for the same value of d_A.

If the equation with the standard parameters gives:

67.27 km / s / Mpc,

then with the varying cosmological constant we will obtain:

$$\frac{H_0}{67.27} = \frac{3.43586}{3.1531} \;\; \rightarrow H_0 \approx 73.30 \;\;\; km/s/Mpc$$

This result would be compatible with the value given by the SN1A but, let us recall that the proposed modification is purely arbitrary and has no physical character[2].

How do we know d_A?

Less us remember that by definition:

$$d_A.\ \theta = d_s$$

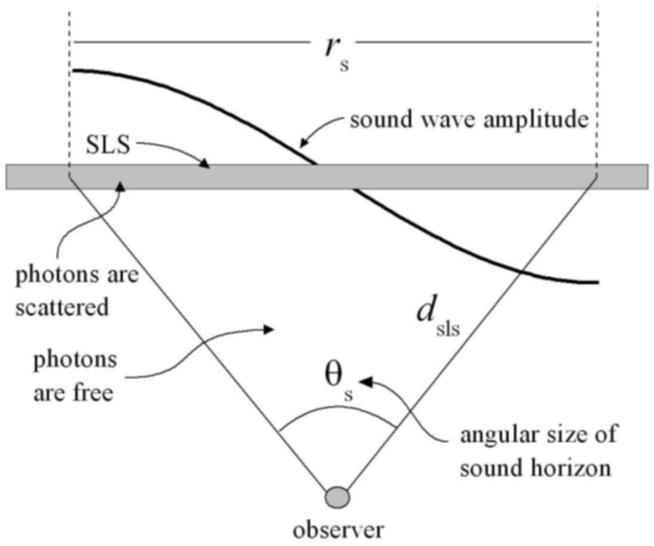
d_A is the angular distance where we see the size d_s of the sonic horizon, under the angle θ which is deduced from the position of the first peak in figure 3, which is equal to:

$$\theta = 0.0104\ radian \approx 0.6\ °,$$

in this case. The size of the object d_s, the sonic horizon, which is the maximum limit for the propagation of sonic waves in the plasma from the origin

$$t = 0 +\ up\ to\ t = 380{,}000\ years$$

up to the photon-matter decoupling, can be deduced from others parameters of the CMB, characterizing the plasma, such as the ratio between the baryons and the photons which are in thermal equilibrium and which determine the sonic speed (velocity of propagation of inhomogeneities in the plasma)[3].



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[2] Note that if it keeps the value of the integral for z = 0, it varies very significantly for 0 <z <1. The value for SN1A would be different with the equation of case b. Let us add that we could modify the function to adjust it even better to the experimental data, it would then remain to provide the physical justification.

[3] The equations giving the size of the sonic horizon are quite tricky, see for instance: https://ned.ipac.caltech.edu/level5/Sept02/Reid/Reid5_2.html,, chapter 5.2, acoustic peaks and the cosmological parameters.

When we, know the size of the horizon [4] and the angle at which we see it, this allows us to define the angular distance d_A and consequently H_0 since we then know all the other parameters.

Conclusion

When the theory in force (the standard model of cosmology) seems to be faulty, before abandoning it for another, assuming that there is, currently, one which is better (this is appreciated on the set of predictions that the theory makes), it must be ensured before that it is used correctly.

The case of cosmology is particular, insofar as, for the theory, drastic simplifying assumptions have been made (homogeneity and isotropy on a large scale), essentially to find analytical solutions! We know that this is highly approximate.

We illustrate how a modification in parameters could change the results. Likely one could do better for matching the experimental data. We selected the cosmological constant, as a sensitive parameter, because it has no well-known physical interpretation.

A plausible hypothesis is that we may be inside a local bubble of ($z = 0.3$ size for instance) where the astrophysical parameters might be different of those (cosmological) of the entire universe. Let us recall that the Ω parameters result from the CMB analysis, a cosmological approach. In this case the theory is not faulty, just the hypothesis retained are not accurate enough for predicting the correct result.

History has shown that an experiment could make a theory falter, as the Michelson-Morley experiment for mechanics which induced special relativity, and that of the black body for mechanics, which induced quantum mechanics.

For the problem of the Hubble constant, if we look at the existing "competing" theories, it is not obvious that there is currently a better one, and some (in particular to quantify gravity) are still under construction, and this despite the considerable research efforts that have been devoted to it. Of course, a theory is not a truth, it is not definitive, and history has shown how they could be improved, a task incumbent on physicists.

This anomaly of the Hubble constant is finally perhaps an opportunity, because by the nature of the problem which it raises, (laws which seem to depend on z, differently from what

[4] How the effect of inflation, linked to a short huge cosmological constant, in the early universe, causing an expansion of a factor exp (100) is taken into account for computing d_s?. If not, it would be necessary to show that for $z>>1$, this is negligible to really validate d_s, otherwise, perhaps the source of the incompatibility lies there...

one thought), like the examples mentioned previously, it can give us information for a lead towards a new approach.

Appendix

The Friedmann-Lemaître equation uses the Robertson-Walker's metric whose coordinates are t, r, θ and φ. As we would like to use the parameter z (the redshift) in place of t, as expansion parameter, because z is an observable, we will calculate the Hubble constant defined in the Friedmann-Lemaître's equation :

$$H = \frac{a'(t)}{a(t)}$$

Where a(t) is the expansion of space factor and a'(t) its time derivative.

$$H = \frac{a'}{a} = \frac{d}{dt}\ln(\frac{a(t)}{a_0}) = \frac{d}{dt}\ln\left(\frac{1}{1+z}\right) = \frac{-1}{1+z}\frac{dz}{dt}$$

By replacing H by its value (for $\Omega_k = 0$), listed below:

$$H = H_0\sqrt{(\,\Omega_{rad}(1+z)^4 \;+\Omega_m(1+z)^3 \;+\Omega_\Lambda)}$$

We get:

$$\frac{dt}{dz} = \frac{-(1+z)^{-1}}{H_0\sqrt{(\,\Omega_{rad}(1+z)^4 \;+\Omega_m(1+z)^3 \;+\Omega_\Lambda)}}$$

This will allow to replace the coordinate time t by the parameter z in the equations.