Cosmology. Hubble constant H_0: What would reconcile measurements made at z >> 1 with those at z <1? Ed.6, 4/10/22

Table of contents

Cosmology. Hubble constant H_0: What would reconcile measurements made at z >> 1 with those at z <1? Ed.6, 4/10 1

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Introduction 2

Brief survey of measurement methods used to determine H_0. 2

Which models are compatible with the experimental data? 3

Luminosity distance 4

Numerical example 7

Deduced value of H_0 8

How do we know d_A? 9

How do we know d_A? 9

Sonic speed of propagation of inhomogeneities in a baryon/photon plasma 10

Calculation of the sonic horizon in the co-moving reference frame 10

The effect of the expansion of the universe 11

.... 13

Case where the propagation of inhomogeneities starts from the end of inflation. 13

How to evaluate the baryon/photon energy density ratio? 14

Which numeric values for the parameters 14

What about the effect of inflation in the calculation of the sonic horizon? 15

Reminder of assumptions 15

Phenomenology of inflation 15

Calculation during inflation 16

Conclusion 17

Appendix 18

Introduction

Currently, this problem of incompatibility, between the values of H_0 at z >> 1 (that using the CMB (Cosmological Microwave Background), giving a value of H_0 of about 67.27 ± 0.6 km / s / Mpc, and those of many methods based on observations at z <1 (SN1A, Cepheids, etc.) giving a value of H_0 of about 73.52 ± 1.62 km / s / Mpc, seems to question the standard cosmological model.

At first, one questioned the accuracy of the observations, but these values deviate by more than 4 σ from their probabilistic diagram so, as so many measurements have been made and verified, this hypothesis is less and less credible.

Today, we are in expectation, fearing for some and hoping for others, a questioning of the standard model and therefore the theory of general relativity which supports modern cosmology. The history of science teaches us that no theory is definitive, but currently, there is not any so effective alternative theories to replace it.

What a theory physically predicts depends on the parameters associated with it and the assumptions made. Let us recall that the assumptions of homogeneity and isotropy of the universe are drastic approximations which, even on a large scale (matter is gathered in filamentous structures with gigantic voids), are far from being really satisfied.

Inhomogeneous and anisotropic models of universes are studied, without too much success so far. We also know that 95% of what generates the dynamics of the universe (dark matter and dark energy) are of unknown nature, despite important research.

On the other hand, the inflation paradigm, which solves a few problems, still looks as an ad hoc theory waiting for some experimental evidences.

. However, before proclaiming the fall of the cosmological standard model, it is worth considering whether it is not its parameters that are wrong.

Here, after a presentation of the problem, on an arbitrary example, we show how the value of H_0 depends on the cosmological parameters. In this document, we assume that the fundamental notions of cosmology are known (Robertson-Walker metric, Friedman-Lemaître equation, density parameters Ω_i, cosmological redshift z, luminosity distance d_L, angular distance d_A, the equation of state cosmological fluids, etc.

If not, and if necessary, see, for instance: http://www.astro.ucla.edu/~wright/cosmolog.htm

Brief survey of measurement methods used to determine H_0.

An essential difference in these methods is the value of the redshift of the observed and measured phenomenon.

Method like SN1A, considered as a standard candle (light source whose intrinsic luminosity is known), for example, where z <1, uses the distance of luminosity d_L deduced from an observable which is the measure of the power of light of the object considered (by a device associated with the telescope measuring the energy of the photon flow) that we will combine with the redshift z (measured by a spectrometer on the telescope), which is another observable of the object considered.

This makes it possible to plot $z(d_L)$ curves for different values of z that we will compare with those predicted by the different models and to eliminate some of them and keep others as possible (best matching method).

Which models are compatible with the experimental data?

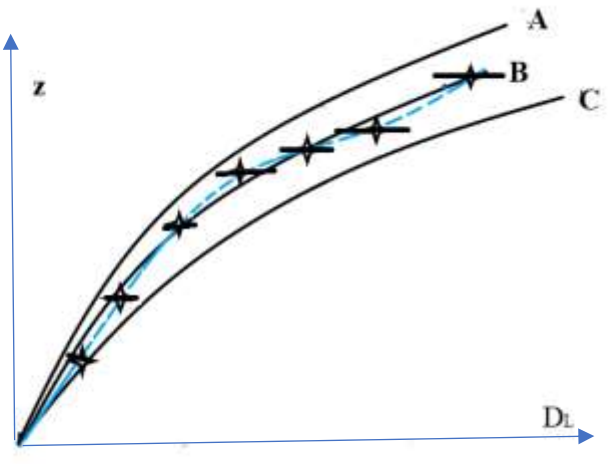


Figure 1: Selection method by best matching to experimental data.

On the diagram above, a set of measurement points for the observations of the redshift z has been represented by star, as a function of the luminosity distance d_L. A dashed curve, connecting them, interpolates the experimental law $z(d_L)$. Each point must be associated with an error bar linked to the inaccuracy of the measurement. We have drawn 3 curves A, B, C corresponding to 3 different cosmological models. We see that for example the curve B is the most compatible with the experimental data. On the other hand, curves A and C are to be excluded.

It is this best fit between observations that will determine which models are compatible with the observations and exclude those that are too far from them, taking into account measurement inaccuracies. Note that this diagram is approximate and only claims to illustrate the phenomenology described.

Luminosity distance

In Euclidean space, the light power F (energy of light received per unit area and per unit of time) from an isotropic source (the Sun for example) of total luminosity L is equal to:

$$F = \frac{L}{4\pi r^2} \qquad (1)$$

where r is the distance between the receiver and the source.

This is simply explained by the fact that the flow is distributed over the surface of a sphere of radius r whose area is $4 \pi r^2$. In a non-Euclidean space, other phenomena must be taken into account.

In an expanding space, as described by the Robertson -Walker metric, the emitted photons will be redshifted by the expansion therefore their energy will be divided by

$$1 + z = \frac{a(t_0)}{a(t_e)} \qquad (2)$$

Where z is the observed redshift of the source and *a (t)* are the scale factors at the emission time of the photon at t_e and at the reception of the photon t_0, (now). A second phenomenon also occurs, the time interval between photons increases all along the geodesic, due to the expansion of space, by a factor also equal to (1 + z), this reduces the received light power.

We will define the luminosity distance d_L, by the relation:

$$F = \frac{L}{4\pi r^2 . a(t_0)^2 (1+z)^2} = \frac{L}{4\pi d_L^2} \qquad (3)$$

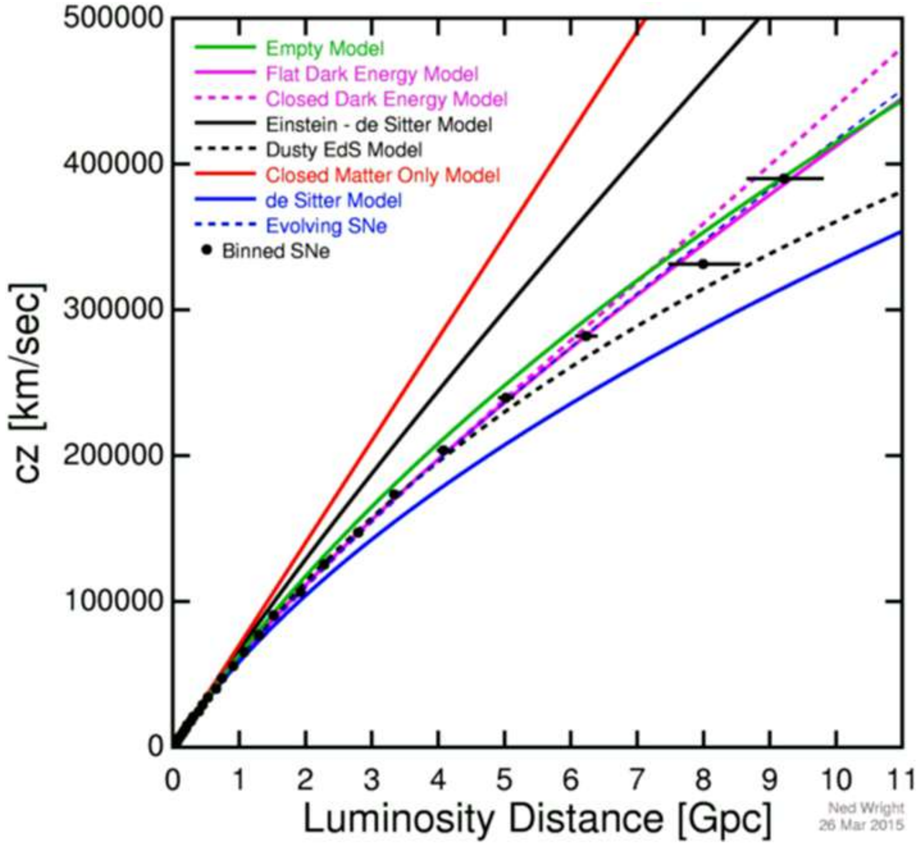
Which gives :

$$d_L = a(t_0) r (1+z) \qquad (4)$$

Note that in equation (3), we do not know r which is a coordinate, but d_L is defined by (3) since L is a standard candle of known absolute luminosity and F is measured by the observer.

Similarly, z is an observable (redshift of the object, measured by a spectrometer).

Let us recall that it is by the diagrams representing the function $z(d_L)$ that the acceleration of the expansion was detected (1998) and that it allowed, as such, to reintroduce the cosmological constant in the standard model, by using the method of the best fit to the experimental data as described in Figure 1,



In addition to discriminating between different cosmological models, the luminosity distance, a simplified expression of which is given by equation (5) below, shows that this luminosity distance depends on the Hubble constant H_0.

It can therefore be used to estimate the value of the Hubble constant. Its general expression is quite tricky, it is simplified if the spatial curvature of the universe is (about) zero, (**$\Omega_k = 0$**).
Let's make this assumption, consistent with the current assumption on spatial curvature.
This luminosity distance d_L from the object located at a spectral shift z * is given by: [1]

$$d_L = \frac{c(1+z)}{H_0} \int_0^{z*} \frac{dz}{\sqrt{\Omega_{rad}(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}} \qquad eq\ 5$$

Note that the expansion parameter is not t but z, an observable. This required a transformation which is described in the appendix. This equation includes H_0, the value of H for $z = 0$, which is a constant in equation 5 (therefore can be outside of the integral). H is linked to H_0 by the formula:

$$H = H_0\sqrt{(\Omega_{rad}(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda)} \qquad eq.6$$

[1] Often, one, for simplifying equations, set c = 1. When necessary or useful, one can resume c in an equation by consistency of dimensional analysis.

Planck and WMAP calculated the value of H_0 by using the Fourier analysis in spherical waves of the inhomogeneities of the CMB which provides the position of the first peak which is an angle which is the value of the angular view size of the sonic horizon. The size of this horizon, in turn, will be calculated by the distance traveled by sonic waves up to the time of baryons/photon decoupling, this depending of the sonic velocity of these waves in the plasma. These two data will provide the angular distance d_A, whose general definition is the distance associated to the viewed angle θ of an object of known size.[2]

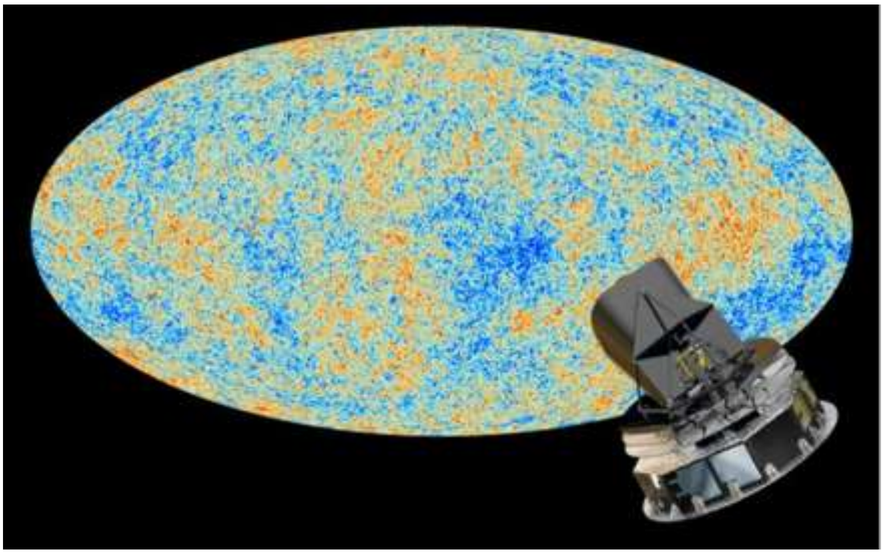


Figure 2: Representation of the temperature of the CMB in "false colors".

The image corresponds to the complete celestial sphere, the Planck Satellite being at the Lagrange L_2 point, 1.5 million km from Earth. All the information is contained in this image.

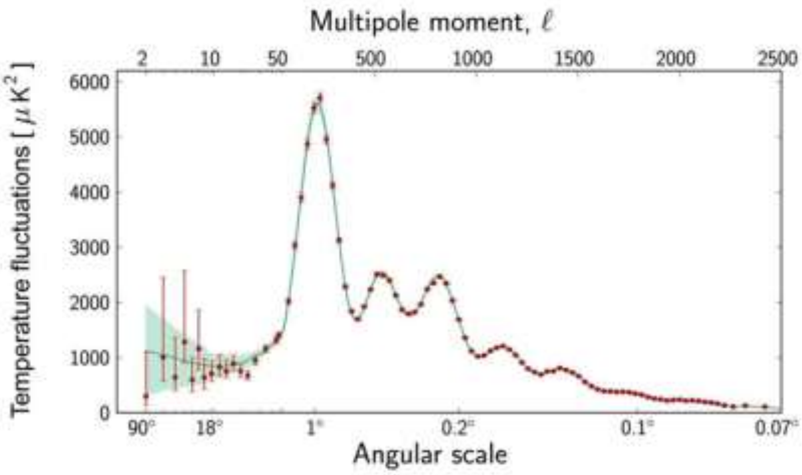


Figure 3: Result of the 2D Fourier transform of the RFC.

[2] As usually θ is a small angle one set $\sin(\theta) = \theta$.

This transform extracts the proportion of patterns associated with each size identified by the value of the multipole or the associated angle.

The plasma being an elastic medium, the inhomogeneities generate acoustic waves, therefore the peaks which correspond to the most intense modes (mainly the fundamental mode, corroborated by its harmonics). This defines the size of the sonic horizon in the plasma.

There is a simple relationship between d_L and d_A:

$$d_A = \frac{d_L}{(1+z)^2} = \frac{c}{(1+z)H_0}\int_0^{z*}\frac{dz}{\sqrt{\Omega_{rad}(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}} \qquad eq\ 7$$

d_A, therefore, also depends on H_0. Let us suppose, for example, that the cosmological constant, instead of being constant, depends on z (and therefore on t). Under these conditions it is necessary to introduce a factor $f(z)$ associated with the cosmological constant λ, which will have a significant effect for $z >> 1$, while keeping the value for $z = 0$. This will introduce a difference of phenomenology.

$$d_A = \frac{c}{(1+z)H_0}\int_0^{z*}\frac{dz}{\sqrt{\Omega_{rad}(1+z)^4 + \Omega_m(1+z)^3 + f(z)\Omega_\Lambda}} \qquad eq\ 8$$

The law that we are going to propose is an example and does not claim any physical character, this will aim to show that the effect can be significant.

In the equation (8), if the integral increases, since the experimental value of d_A is a given experimental data [3], the factor H_0, must increase for keeping d_A constant.

The exact impact of this correction is likely quite complex, but an empirical example modifying the vacuum equation of state of the cosmological constant is given as an example to illustrate the mechanism.

Numerical example

We are interested in the case z >> 1 (method using the results of Planck which are more recent than those of WMAP).

$\Omega_m = 0.306$

$\Omega_{rad} = 0.00009236$

$\Omega_\Lambda = 0.694$

[3] In fact, dA is deduced from the analysis of CMB and cosmological parameters. The size of the "sonic" horizon is deduced from the time until decoupling and from the velocity of sound (velocity of propagation of inhomogeneities in a photon-baryon plasma) and the angle of sight is given by the Fourier transform of the CMB (position of the first peak).

Let's write the part of the equation that describes the influence of H_0 with these conventions and values:

a) Case where the cosmological constant does not vary

$$\frac{1}{H_0}\int_0^{1089} \frac{dz}{\sqrt{0{,}00009236(1+z)^4 \ +0{,}306(1+z)^3 \ +0{,}694}} \qquad eq\ 9$$

b) Case where the cosmological is no longer constant and is a function of z while keeping its value for $z = 0$.

Let us say, for example

$$f(z) \ = \ 10^{-z^{10^{-3}}}$$

which satisfies these constraints.

$$\frac{1}{H_0}\int_0^{1089} \frac{dz}{\sqrt{0{,}00009236(1+z)^4 \ +0{,}306(1+z)^3 \ +(0{,}694)10^{-[z^{(10^{-3})}]}}} \qquad eq\ 10$$

a) In this case, the value of the integral given by mathematica's "NIntegrate" (numerical integration) function is : 3.15393

b) In this case, the result given by mathematica by the function NIntegrate is : 3.43586

Deduced value of H_0

Let us Recall the equation that governs these parameters

$$d_A = \frac{c}{(1+z)H_0}\int_0^{z*} \frac{dz}{\sqrt{\Omega_{rad}(1+z)^4 \ +\Omega_m(1+z)^3 + f(z).\Omega_\Lambda}}$$

We see that if we know d_A and the value of the integral, since we know the redshift z and as c is the velocity of light, we can deduce H_0. If the value of the integral increases the value of H_0 (which is the denominator) must also increase for the same value of d_A.

If the equation with the standard parameters gives:

67.27 km / s / Mpc,

then with the varying cosmological constant we will obtain:

$$\frac{H_0}{67.27} = \frac{3.43586}{3.1531} \ \rightarrow H_0 \approx 73.30 \ \ km/s/Mpc$$

This result would be compatible with the value given by the SN1A but, let us recall that the proposed modification is purely arbitrary and has no physical character.

How do we know d_A?

Less us remember that by definition:

$$d_A.\ \theta = d_s$$

d_A is the angular distance where we see the size d_s of the sonic horizon, under the angle θ which is deduced from the position of the first peak in figure 3, which is equal to:

$$\theta = 0.0104 \text{ radian} \approx 0.6\ °,$$

in this case. The size of the object d_s, the sonic horizon, which is the maximum limit for the propagation of sonic waves in the plasma from the origin

$$t = 0 + \text{ up to } t = 380{,}000 \text{ years}$$

up to the photon-matter decoupling, can be deduced from others parameters of the CMB, characterizing the plasma, such as the ratio between the baryons and the photons which are in thermal equilibrium and which determine the sonic speed (velocity of propagation of inhomogeneities in the plasma)[4].

When we, know the size of the horizon and the angle at which we see it, this allows us to define the angular distance d_A and consequently H_0 since we then know all the other parameters.

How do we know d_A?

By setting $d = d_s$ for marking that it is a sonic horizon, let us recall that:

$$d_A.\ \theta = d_s \qquad \text{eq 12}$$

d_A is the angular distance where we see, in a curved spacetime, the size d_s of the sonic horizon, under the angle θ, which is deduced from the position of the first peak as shown in figure 3.

In our universe $\theta = 0.0104$ radian $\approx 0.6°$.

The size of the object d_s, which is the sonic horizon, is the maximum limit of propagation of the sonic waves in the plasma from the origin $t = 0+$ up to $t = 372000$ [5] years (at photon-matter decoupling). It is deduced from other parameters of the CMB, characterizing the plasma, such

[4] The equations giving the size of the sonic horizon are quite tricky, see for instance:
Ref A https://ned.ipac.caltech.edu/level5/Sept02/Reid/Reid5_2.html,, chapter 5.2, acoustic peaks and the cosmological parameters
. See also .Ref B : https://physics.stackexchange.com/questions/450517/how-did-the-planck-study-calculate-the-angular-size-of-the-sound-horizon for an interesting contribution, partly related here.…

[5] You may also find $t = 380000$ years in some papers.

as the ratio between the baryons and the photons which are at thermal equilibrium and which determine the sonic speed (speed of propagation of inhomogeneities in the plasma).
Let us start by recalling some topics for understanding how d_s was calculated, from the data on the cosmological parameters of the CMB, collected by WMAP and Planck, for instance.

Sonic speed of propagation of inhomogeneities in a baryon/photon plasma

The speed of propagation (celerity) of a disturbance (inhomogeneity of the plasma) in a plasma is denoted c_s.

Calculation of the sonic horizon in the co-moving reference frame

$$d_s = \int_{0+}^{t_{CMB}} c_s\, dt \quad = \int_{0+}^{t_{CMB}} \frac{c.dt}{\sqrt{3(1+\frac{3\rho_B(1+z)^3}{4\rho_\gamma(1+z)^4})}} \quad \sim \int_{0+}^{t_{CMB}} \frac{c.dt}{\sqrt{3}} \quad eq\ 13$$

In the equation (13) whose comparison between the 2nd term and the 3rd shows the value of c_s. The size d_s is given by integrating the function from the time $t = 0+$ up to the time of the decoupling t_{CMB}, of this expression, where ρ_B is the baryon density and ρ_γ the photon density, parameters which take into account the relative number of baryons compared to photons, but also their individual energy, for example expressed in eV. The last term of equation (13) is an approximation when $\rho_\gamma >> \rho_B$.

It is known that baryons own energy is about 1 GeV, for photons it depends on the "temperature" of the universe. Thus, upon decoupling, the photons have an energy of about 0.26 eV, but since there are one billion per baryon, the order of magnitude of the ratio is fairly not too far of 1.

On the other hand, at t = 1 second, the energy average photon is 1 MeV so, per their overwhelming number, the fraction in the square root tends to zero, which simplifies the equation.

In this period from t = 0+ to t_{CMB}= 372,000 years there is a first phase dominated by radiation then a second phase where matter (baryons) dominates. The turning point is around t = 60,000 years for z = 3000, at a temperature of 8000K.

We see that, as in most of the time of the period the energy density of the photons is fairly higher than that of the baryons, the simplified formula can be used as a first approximation.

It is simply calculated and for t_{CMB} = 372,000 years we get:

$$d_s = \int_{0+}^{t_{CMB}} \frac{c.dt}{\sqrt{3}} = t_{CMB} \frac{c}{\sqrt{3}} \sim 0.066\, Mpc \qquad eq\ 14$$

Of course, in the Planck collaboration, rigorous, more complex calculation prevailed. But here, as our goal is to show the principle of the calculation, the approximation makes it possible to illustrate it numerically.

With these data ds = 0.066Mpc and θ = 0.0104 radian, in the co-moving reference frame, we gett: d_A = ds/θ = 6.346 Mpc, i.e. about half of the calculation made by the Planck collaboration.

The reason is that the equations above give the size in a co-moving reference frame but it must be taken into account that the universe is expanding.

The effect of the expansion of the universe

To take into account the fact that during the propagation of sonic waves the universe was expanding, we use the following equations:

$$d_s = \int_0^{t_{CMB}} \frac{c_s\, da}{a(t)} = \int_0^{a_{CMB}} \frac{c_s da}{H\, a^2} \sim \frac{c_s}{\sqrt{\Omega_m}\, H_0} \int_{z_{CMB}}^{\infty} \frac{dz}{(1+z)^{3/2}} \qquad eq\ 15$$

In this equation, proposed in reference B of note 5, we make the assumption of a universe dominated by matter at decoupling, c_s is the assumed constant sonic speed, CMB the time of decoupling matter radiation, the other parameters have already been defined.
We recall that da/dt = a.H. This is used to go from the first integral to the second.

$$H \approx H_0\, \Omega_m^{1/2}\, (1+z)^{3/2} = H_0\, \Omega_m^{1/2}\, a^{-3/2}$$

See equation (2) where, as we set $a_0 = 1$, then $(z+1) = a_0/a = 1/a$.
This also means that dz = -da/a2, the "minus" sign implying the permutation of the integration limits of the last integral.
Note that the last term of equation 15 is very approximate, because we assumed the constant speed c_s which is not the case and also a universe dominated by matter, which is only true from z < 3000. The goal is to allow by a simple calculation to give orders of magnitude.

We will give a more accurate version in a further paragraph.
For the parameters of the problem and assuming that the sonic velocity is $c.3^{-1/2}$, this gives:

$$d_s = \frac{2\,c}{H_0\sqrt{3\Omega_m}}(1+z_{CMB})^{-1/2} \qquad eq\ 16$$

For H_0 = 70km/s/Mpc, Ω_m=0.3 and z_{CMB} = 1090, this gives approximately 270 Mpc, which must be divided by (1+ z_{CMB}) to insert it into the angular distance term of the calculation.

This gives an angular diameter of 0.019 radians, already closer to the value given by the Planck collaboration.
But if the speed of sound is lower then the scale decreases. Recall that the speed of sound is given by the equation:

$$c_s = \frac{c}{\sqrt{3(1+3\rho_B\,/4\rho_\gamma)}} \qquad eq\ 17$$

We see that the baryon to radiation ratio increases with time, in proportion to the factor a(t).
At decoupling the ratio $3\rho_B\,/4\rho_\gamma \approx 1$, which leads to:

$$c_s\,(t_{CMB}) = \frac{c}{\sqrt{6}} \qquad eq\ 18$$

This leads to a correction that decreases the size of the sonic horizon towards the size predicted by the Planck collaboration, which has treated the problem rigorously.

This qualitative presentation, was aimed to focuses on showing the phenomena at work in this problem, because we feared that the mathematical complexity of the more rigorous solution would blur these phenomena for the layman.

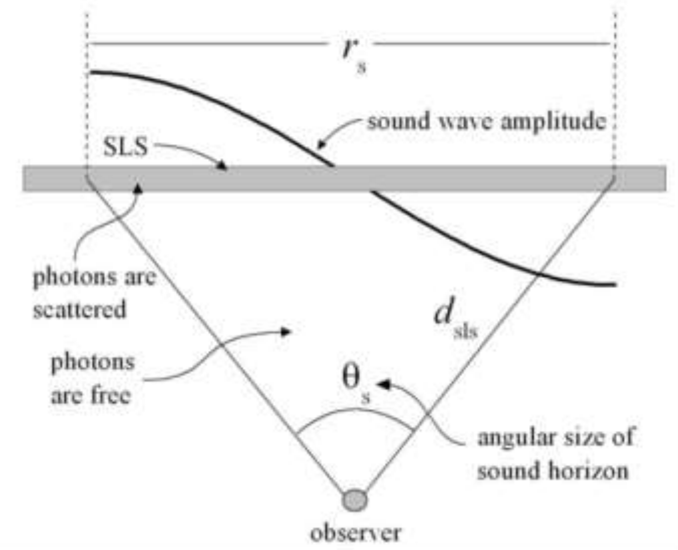


Figure 4: A representation of the angular distance calculation

Knowing the size of the horizon and the angle under which we see it, this allows us to define the angular size d_A and consequently H_0 since we then know all the other parameters.

Case where the propagation of inhomogeneities starts from the end of inflation.

Here, it is assumed that inhomogeneities of macroscopic size result from inflation and that it is not relevant to consider them before its end. Let's start by giving a more accurate version of the last term of equation 15.

$$d_s = \frac{1}{H_0}\int_{z_{CMB}}^{z_{INF}} \frac{c.}{\sqrt{3(1+\frac{3\rho_B}{4(1+z)\rho_\gamma})}} \frac{dz}{\sqrt{\Omega_m(1+z)^3+\Omega_\gamma(1+z)^4}} \quad eq\ 19$$

In this equation we assumed that the universe is without curvature (Ω_k =0), that the cosmological constant was negligible during this era, taking into account only matter and radiation.

The integration is done from the CMB at the end of inflation.
Note also that in the equation we have the density of the baryons which intervenes in the plasma (dark matter does not intervene because it does not couple with the radiation) but also the density parameter of the total matter (baryons + matter black).

We should not confuse these 2 parameters.

How to evaluate the baryon/photon energy density ratio?

We know that there are about 1 billion photons per baryon. The energy of baryons is mainly their rest mass energy, around 1 GeV, their kinetic energy remains low compared to that, and when approaching very high energy > 1 GeV where they begin to be relativistic, photons in overwhelming numbers are also at this energy, which means that the baryon energy / photon energy ratio remains very low.

Which numeric values for the parameters

The equilibrium point between the radiative era and the material era (which includes dark matter) is around z = 3000 where the temperature is around 8000°K, i.e. 0.75 eV for photons.

In equation 19 let us assume that for z = 3000

$$\frac{3}{4(3000)}\frac{\rho_B}{\rho_\gamma}\sim 1 \rightarrow \frac{\rho_B}{\rho_\gamma}\sim 4000 \quad eq\ 20$$

Substitute this result, together with the known values of the Ω, into equation 19

$$d_s = \frac{c}{H_0}\int_{z_{CMB}}^{z_{300000}} \frac{1}{\sqrt{3(1+\frac{3000}{(1+z)})}}\frac{dz}{\sqrt{0.3(1+z)^3+0.0000924(1+z)^4}} \quad eq\ 21$$

This integral can be integrated from z = 1090 to z = 300,000.
Beyond a value of z = 300,000 for example, this ratio will be considered to be zero, which simplifies the calculations as we use the simplified form, for z > 300,000 and below a more complex form that we will integrate.

$$d_{s2} = \frac{c}{H_0}\int_{z_{300\,000}}^{z_{INF}} \frac{dz.(1+z)^{-2}}{\sqrt{3*0.0000924}} = [\frac{c\,(1+z)^{-1}}{H_0\,(0.01665)}]\,_{z=inflat}^{z=300000} eq\ 22$$

What about the effect of inflation in the calculation of the sonic horizon?

In this analysis, it appears that the inflationary phase, whose expansion equations are different where one can wonder what the state of the plasma was, is not taken into account. Is it because it leads to unreasonable complexity or because it was assumed that their influence would be negligible? Yet inflation induces a significant difference in phenomenology.

Reminder of assumptions

The equations presented invoke the cosmological time t (in Robertson-Walker metric) and the scale factor a(t), which are related by relations $a(t) = kt^{1/2}$ for the photon era (dominated by photon energy in the plasma) or baryonic $a(t) = Kt^{2/3}$, for the era dominated by matter (from t = approximately 60,000 years). The proposed equations giving the sonic velocity take into account a transition which is gradual. Under these assumptions, these equations are assumed to be valid from t = 0+ where a(t) = 0+ to $t = t_{CMB}$ where $a(t_{CMB}) = a(t_0)$ /1089, where t_0 is "today".

Phenomenology of inflation

The inflation period is characterized by a function of the scale factor a(t) of the type $a(t) = e^{H.t}$, where H is the value of the Hubble constant at the start of inflation which remains constant throughout the period of inflation. Generally located between $t_{beginning} = 10^{-30}$s and $t_{end} = 10^{-28}$ s (in cosmological time), which means that this phase lasted 100 times the age of the universe at the beginning of the phase.

The expansion (on a space dimension) is therefore proportional to $e^{H.100}$. As $e^{100} \approx 10^{43}$ we see that the expansion has been gigantic. If we compare this value to that which would result from the expansion without inflation which, in a universe dominated by radiation, follows a law proportional to $t^{1/2}$, this would have given an expansion of 10.

The difference is enormous. Then, if we calculate the variation of a(t), between the end of inflation $t = 10^{-28}$ s and the decoupling (t = 372,000 years = $1.173\ 10^{13}$ s) by approximating using the formula $a(t) = k.\ t^{1/2}$, we obtain approximately a variation less than 10^{-21}, i.e. 10^{22} times smaller than that of inflation. Even if a(t) is tiny, can we neglect inflation in such circumstances, since the variation of a(t) was almost entirely the work of inflation?

Moreover, it was during the period of inflation that the macroscopic inhomogeneities developed, another parameter that must be considered, but again with caution, because this process (dilation of quantum fluctuations) took place throughout this inflation phase giving a nearly scale-invariant power spectrum.

If this last remark would lead to consider the phenomenon of propagation of inhomogeneities in the plasma from the end of inflation $t = 10^{-28}$ s, or at least from a certain stage in the inflation (before they existed only at the microscopic scale, even if it was in a very small universe), a calculation of the effect of inflation, on the phenomenon of propagation of inhomogeneities in a plasma, inflation which moreover had essential structural consequences on other phenomena, is welcome, taking into account the problem of incompatibility of the result of H_0 given by this method with that for $z < 1$.

Calculation during inflation

We must calculate H at the end of inflation, As H is constant during inflation this will give its value. To do this, z must be calculated at the end of inflation. Knowing that at t = 60,000 years, z = 3000, let's calculate the variation of z until $t = 10^{-28}$s. knowing that we are in the radiative era where $a(t) = k.t^{1/2}$, (varies according to the square root of t). For *t* the variation is 60,000 years ($1.89\text{x}\ 10^{12}$ s), multiplied by $10^{28} = 1.89.10^{40}$. The value of z at the end of inflation is then: $z_{inf} = 3000\ \text{x}1.37\text{x}\ 10^{20} = 4.1\text{x}\ 10^{23}$. Calculate H, at the end of inflation using equation 6 for $z = 4.1\text{x}\ 10^{23}$

$$H = H_0\sqrt{(\ \Omega_{rad}(1+z)^4\ + \Omega_m(1+z)^3\ + \Omega_\Lambda)}$$

$$H = 67{,}27\sqrt{(0.00009246(4.1\ 10^{23})^4\ + 0.3(4.1\ 10^{23})^3\ + 0.694)}\ \sim 1.087\ \ 10^{47}$$

We will report this value in the simplified equation where cs is constant but where the expansion of the universe is given by $a(t) = e^{Ht}$ which is the solution of the equation $H = a'(t)/a(t) = H$ = constant.

The relation between z and a(t) is $z+1 = a_0/a(t) = 1/a(t)$ by setting $a_0=1$, because a_0 is the scale factor today which is the reference. Therefore:

$$d_{s-inf} = c\int_{t=0}^{t=100} \frac{1.}{\sqrt{3}}\frac{dt}{a(t)} = c\int_{t=0}^{t=100} \frac{1.}{\sqrt{3}}\frac{dt}{e^{Ht}} = [\frac{c}{\sqrt{3}}(\frac{e^{-Ht}}{H})]_{t=10^{-30}}^{t=10^{-28}}$$

In this equation we used the integral of e^{-Ht} which is $-e^{-Ht}/H$

$$d_s = \frac{c}{\sqrt{3}}\left(\frac{e^{-H.10^{-28}}}{H}\right)(1-e^{100}) = \frac{c}{\sqrt{3}}\left(\frac{2.44\ 10^{-24}}{1.087\ 10^{47}}\right)(1-e^{-100})$$

We replaced the expression of $a(t) = e^{Ht}$ by its value ($2.44\ 10^{-24}$) at the end of inflation (at 10^{-28}s). With the huge value of H at the time of inflation, and its presence in the negative exponential, this expression yields a negligible result. We have not solved the problem of incompatibility between measurements of the Hubble constant for z < 1 and those for z >> 1.[6]

Conclusion

When the theory in force (the standard model of cosmology) seems to be faulty, before abandoning it for another, assuming that there is, currently, one which is better (this is appreciated on the set of predictions that the theory makes), it must be ensured before that it is used correctly.

The case of cosmology is particular, insofar as, for the theory, drastic simplifying assumptions have been made (homogeneity and isotropy on a large scale), essentially to find analytical solutions! We know that this is highly approximate.

History has shown that an experiment could make a theory falter, as the Michelson-Morley experiment for mechanics which induced special relativity, and that of the black body for mechanics, which induced quantum mechanics.

For the problem of the Hubble constant, if we look at the existing “competing” theories, it is not obvious that there is currently a better one, and some (in particular to quantify gravity) are still under construction, and this despite the considerable research efforts that have been devoted to it. Of course, a theory is not a truth, it is not definitive, and history has shown how they could be improved, a task incumbent on physicists.

[6]Let us stress that this document should be considered as an exercise No doubt that the Planck collaboration carefully analyzed the problem leaving little opportunity for oblivion in such process.

This anomaly of the Hubble constant is finally perhaps an opportunity, because by the nature of the problem which it raises, (laws which seem to depend on z, differently from what one thought), like the examples mentioned previously, it can give us information for a lead towards a new approach.

Appendix

The Friedmann-Lemaître equation uses the Robertson-Walker's metric whose coordinates are t, r, θ and φ. As we would like to use the parameter z (the redshift) in place of t, as expansion parameter, because z is an observable, we will calculate the Hubble constant defined in the Friedmann-Lemaître's equation :

$$H = \frac{a'(t)}{a(t)}$$

Where a(t) is the expansion of space factor and a'(t) its time derivative.

$$H = \frac{a'}{a} = \frac{d}{dt}\ln(\frac{a(t)}{a_0}) = \frac{d}{dt}\ln\left(\frac{1}{1+z}\right) = \frac{-1}{1+z}\frac{dz}{dt}$$

By replacing H by its value (for $\Omega_k = 0$), listed below:

$$H = H_0\sqrt{(\,\Omega_{rad}(1+z)^4 \; + \Omega_m(1+z)^3 \; + \Omega_\Lambda)}$$

We get:

$$\frac{dt}{dz} = \frac{-(1+z)^{-1}}{H_0\sqrt{(\,\Omega_{rad}(1+z)^4 \; + \Omega_m(1+z)^3 \; + \Omega_\Lambda)}}$$

This will allow to replace the coordinate time t by the parameter z in the equations.