Cosmology. Problem of the Hubble constant H_0: a way to reconcile the measurements made by PLANCK and WMAP (phenomenon observed at z >> 1 with those at z <1, such as the SNIA method, for instance).

Introduction

Currently this problem of incompatibility between the values of the Hubble constant, which value is H_0, now days, deduced from the measurements by methods based on observations at z >> ,1 such as that of the CMB (Cosmological Microwave Backgroung), giving a value of H_0 of approximately 67.27 ± 0.6 km / s / Mpc, and those of many other methods based on observations at z <1 (SN1A, Cepheids, etc.) giving a value of H_0 of approximately 73.52 ± 1.62 km / s / Mpc, seems to question the standard cosmological model.

We may question the accuracy of the observations, but these values deviate by far more than 4 σ from their probabilistic diagram therefore, this hypothesis is less and less credible.

Up today, we are in the doubt, some fearing and others hoping that this would invalidate the standard model and even of the theory of general relativity, itself, of which we know that, if it certainly has nothing definitive as the history of science suggests, there is not much available and so effective other theory for replacing it. What a theory physically predicts depends on the parameters associated with it and the assumptions made.

Let us recall that the assumptions of homogeneity and isotropy of the universe are drastic approximations which even on a large scale (matter is gathered in filamentous structures with gigantic voids) are far from being really satisfied. Inhomogeneous and anisotropic models of universes are studied, without much success so far. We also know that 95% of what generates the dynamics of the universe (dark matter and dark energy) are of unknown nature, despite important research. Moreover, the paradigm of inflation, while it advantageously solves a few problems, is still an ad hoc theory. Some physicists believe that we will be able to provide experimental proof, which would strengthen the hypothesis, but for the moment such proof is lacking. With this problem on the Hubble constant, it's starting to do a lot for this standard model of cosmology.

However, before discarding definitely the general relativity, it is worth considering how we use the parameters. Here we propose a method which yields a result different of that of the usual calculation. General relativity, of which cosmology is an application, is a (geometric) theory of gravitation, it only deals with the gravitational interaction and does not take into account the 3 other interactions at all, two of which are very local but of which the another which is electromagnetism is also a long-range interaction (reputed to be infinite). If light, photons, participate as a fluid, in the dynamics of the universe, it is by using their energy-momentum tensor which is symmetrical. The electromagnetic interaction is governed by the electromagnetic tensor which is an antisymmetric tensor (these tensors are related but rule very different phenomenology's). Thus, as we will develop in this document, in the plasma era, the coupling, via the electromagnetic interaction, between the charged matter of the plasma and the photons can influence the dynamics. In this method, it is not relativity which was faulty, but simply the analysis of the problem.

In this document, we assume that the fundamental notions of cosmology, (Robertson-Walker metric, Friedman-Lemaître equation, density parameters Ω_i, cosmological spectral shift z, luminosity distance D_L, angular distance D_A, equation of state cosmological fluids, etc., are known by the reader. If this is not the case and if necessary, see the tutorial:

http://www.astro.ucla.edu/~wright/cosmolog.htm

Brief survey of measurement methods used to determine H_0.

An essential difference in these methods is the value of the spectral shift (z) of the observed and measured phenomenon.

Methods like, for instance, methods using the SN1A and Cepheids, which are objects at z <1, are supposed to be standard candles (light sources whose intrinsic power are known), use a parameter called the distance of luminosity D_L.

This distance of luminosity is deduced from an observable which is the measurement of the received power of light of the considered object (by a detector in the telescope measuring the energy of the photon flux). We will associate it with the spectral shift z (measured with a spectrometer in the telescope), which is another observable of the same object. This will make possible to plot z = f (D_L) curves for different values of D_L, that we will compare with those predicted by the different models, and to eliminate some of them and keep others as possible, (best fit method).

****

Which cosmological models are compatible with experimental data?

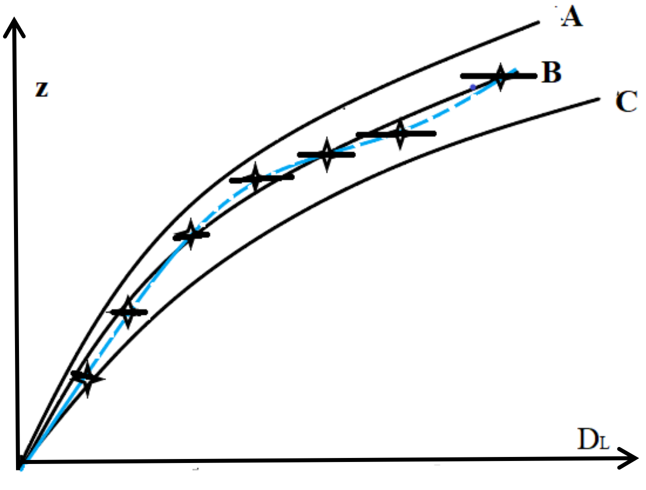

Figure 1: Best fit method for selection of cosmological models

On the diagram above, a set of measurement points for the observations of the spectral shift z, has been represented by stars, as a function of the luminosity distance D_L. A dashed curve, connecting them, interpolates the experimental law z (D_L). Each point must be associated with an error bar linked to the inaccuracy of the measurement. We have drawn 3 curves A, B, C corresponding to 3 different cosmological models. We see that, for example, curve B is the most compatible with the experimental data. On the other hand, curves A and C are to be excluded.

It is by this best fit method between the observations curve and those of cosmological models that we will select which models are compatible with the observations and exclude those that are too far from them, considering measurement inaccuracies. Note that this diagram is approximate and only claims to illustrate the phenomenology described.

****

Considering that the measured spatial curvature of the universe is zero, ($\Omega_k = 0$), as this simplifies the calculations, we will make this assumption, in our calculations. The distance of luminosity D_L of the object located at a spectral shift z * is given by:

$$D_L = \frac{1+z}{H_0} \int_0^{z*} \frac{dz}{\sqrt{\Omega_{rad}(1+z)^4 \; + \Omega_m(1+z)^3 \; + \Omega_\Lambda}} \qquad eq\,1$$

Note that the dynamic parameter is z, an observable. This required a transformation which is described in the appendix. This equation includes a term which is the value of H (its value varies from that of H_0, for z = 0, up to the maximum value of H, for the spectral shift z *, at the cosmological time t* of the observed phenomenon. As H_0 is a constant, in equation 1, it can be extracted from the integral.

$$H = H_0\sqrt{(\,\Omega_{rad}(1+z)^4 \; + \Omega_m(1+z)^3 \; + \Omega_\Lambda)} \qquad eq.2$$

The method used by Planck and WMAP extracts the value of H_0 from a Fourier analysis in spherical waves of the inhomogeneities of the CMB. The first peak of it gives the value of the angular size of the horizon in angle (more often we use moments, but there is a correspondence between the two).

This corresponds to another observable which is the angular distance D_A, which relates the angular size θ of an object of known intrinsic size D to the distance by the relation: $\theta . D_A = D$.

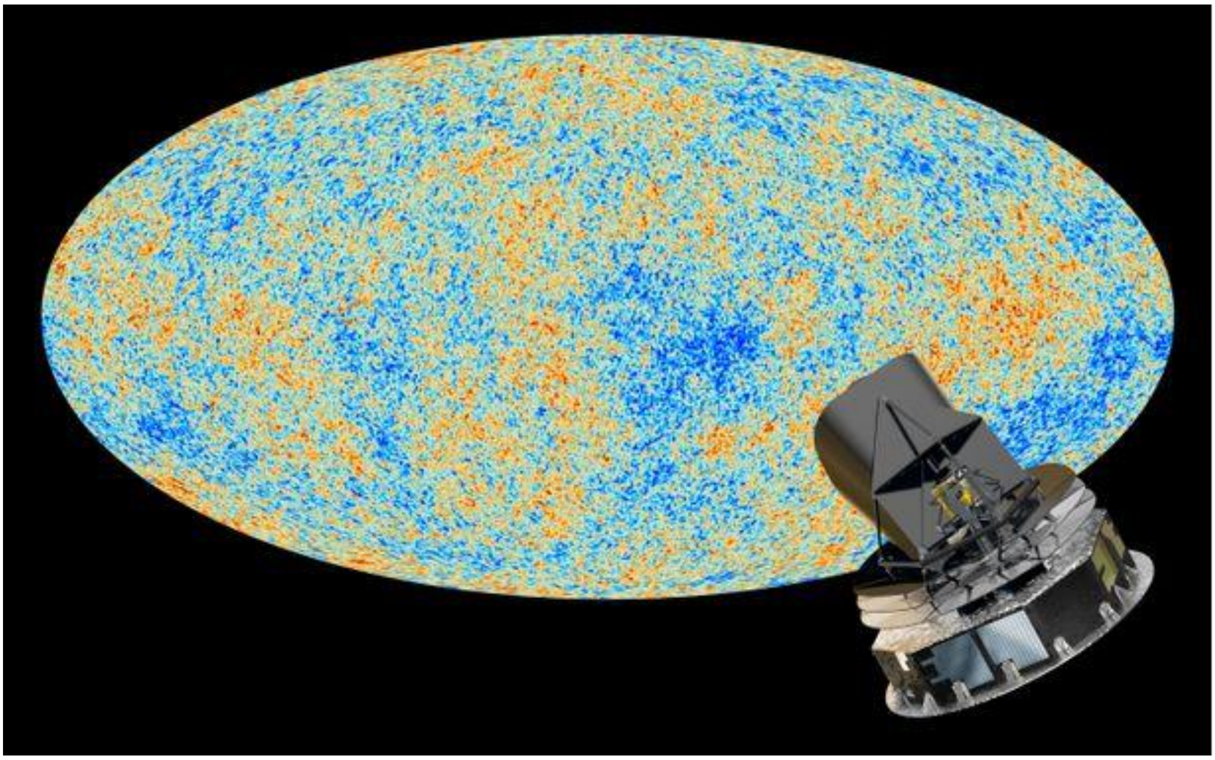

Figure 2: CMB image, in false colors representing the temperature of inhomogeneities.

The image above corresponds to the full celestial sphere, The Planck Satellite being at the Lagrange L2 point. As this point is at 1.5 million km from the Earth, he sees the entire celestial sphere (360 °) from its “center”. This is difficult to represent, but this is what the image is trying to suggest. The ovoid shape of the image results from the projection of the celestial spherical surface onto a plane, which cannot be achieved without distortion.

Result of the Fourier 2D transform process

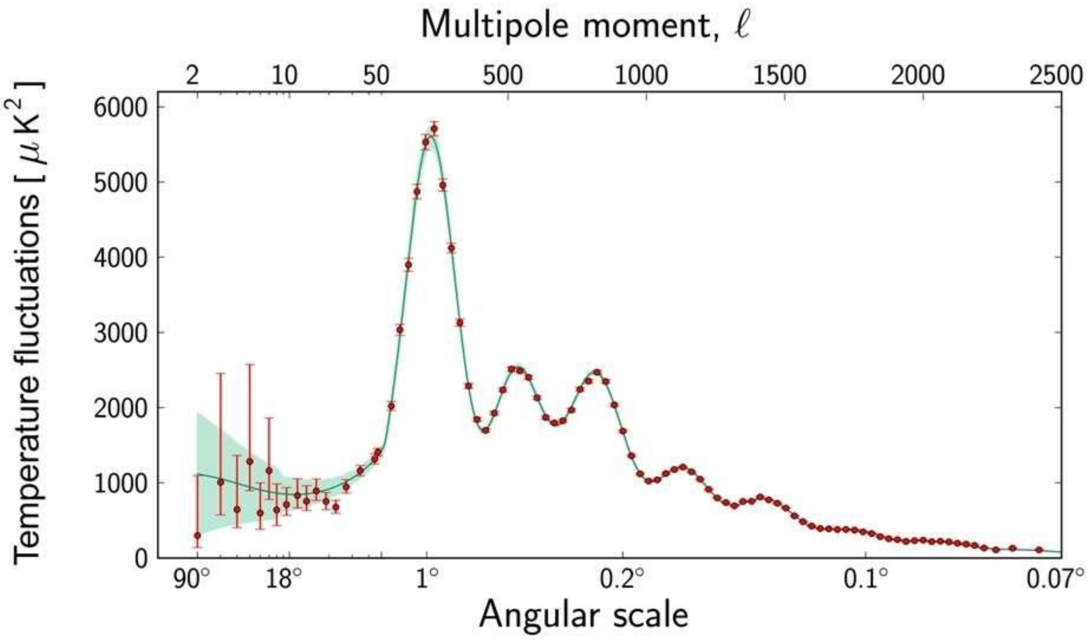

Figure 3: Result of the 2D Fourier transform decomposition of the CMB.

This decomposition makes it possible to extract the ratio of patterns of each size of inhomogeneity, identified by the value of the multipole or its associated angle. The plasma being an elastic medium, the inhomogeneities generate acoustic waves. Therefore, the peaks correspond to the most intense modes (mainly the fundamental mode, corroborated by its harmonics). This characterizes the size of the plasma (its horizon).

A phenomenology like that of vibrating strings, and acoustic resonances of sounds propagating in the air, in a closed box. There is a simple relationship between D_L and D_A:

$$D_A = \frac{D_L}{(1+z)^2} = \frac{1}{(1+z)H_0}\int_0^{z*} \frac{dz}{\sqrt{\Omega_{rad}(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}} \qquad eq.3$$

Therefore, D_A depend on H.

Can we attribute the same equation of state to dark matter and baryonic matter?

In equation 1, dark matter and baryonic matter are not separated. The density parameter Ωm includes both. This assumes that they have the same equation of state, which implies that their "dynamics" are governed by the same factor $(1 + z)^3$.

This is certainly acceptable for z <1, but for z >> 1 it would be surprising if this would be still the case.

Indeed, before the decoupling z > 1089, the universe is a plasma where the dark matter (in majority in the matter) does not interact with the photons of the plasma, this allowing to preserve the main part of the inhomogeneities. The baryons (mainly the protons) have an electric charge and therefore interact strongly, via the electromagnetic interaction, with the huge number of photons in this plasma (approximately 1 billion photons per baryon). This plasma is at a quasi-thermal equilibrium (the fluctuations that we measure are of the order of 10^{-5}).

What one observes on the "last surface of diffusion: the cosmological background radiation (CMB)", it is the result of what occurred during all the plasma era where 2 phenomena were opposed:

The dark matter freezing the fluctuations of the plasma and the baryons, per their interaction with the huge number of photons, smoothing the fluctuations. As dark matter is dominant, most of the fluctuations have been preserved, but if there had been only baryons, what we would observe today would have been quite different: the fluctuations would be even weaker than they are.

Therefore, for z >> 1, we have two different kinds of phenomenology, one for dark matter and another for baryonic matter, this being is negligible for z <1.

Under these conditions it seems reasonable to consider them as two different kinds of matter with different equations of state and this can affect the value of H for z >> 1, therefore, on the value of H_0, that we deduce from it, by the equation.

$$D_A = \frac{1}{(1+z)H_0} \int_0^{z*} \frac{dz}{\sqrt{\Omega_{rad}(1+z)^4 + \Omega_{dm}(1+z)^3 + \Omega_{bar}(1+z)^3 f(z) + \Omega_\Lambda}}$$

In this equation we have split Ω_m into Ωd_m and Ω_{bar} and we have introduced a coupling factor f (z) for the baryons, the effect of which must be negligible for z <1.

What effect can this difference in phenomenology, generate?

In a thermal equilibrium, it is the number of "free" particles that counts. But if particles are coupled, they can no longer be considered as 2 free particles but rather behave as only one.

This will make decrease the number of free baryons participating at thermal equilibrium, which, in turn lowers the density parameter of baryonic matter. Note that for the photons coupled with baryons as there are one billion per baryon, the decrease is negligible.

The angular distance D_A is a measured parameter therefore known, as well as the spectral shift z which is also a measured parameter. This defines the factor H.

If the part, in parentheses of the right-hand-side of equation 1 decreases, as the value of H does not vary, the factor H_0, must increase. Therefore, the measurement made from the CMB will be closer to those of measurements made by methods using sources at $z < 1$.

This goes in the right direction to reconcile the two deemed incompatible measures.

The exact impact of this correction is undoubtedly quite complex, but an empirical example separating dark matter from baryons is given as an example to illustrate the mechanism.

Empirical example of the effect of taking this remark into account.

The following example is purely illustrative of the effect of assigning different dynamic to dark matter and baryons, due to their different equation of state, for showing how this affects the value of H_0.

For dark matter, the density parameter is Ω_{dm} and for baryonic matter the density parameter is Ω_{bar}.

We will model the coupling of baryonic matter (which causes Ω_{bar} to decrease when z increases) by a function which has a negligible effect for $z < 1$ and only has a sensible effect for $z >> 1$.

We are interested in the case $z >> 1$ (method using the Planck results which are more recent than those of WMAP).

$\Omega_{dm} = 0.258$

$\Omega_{bar} = 0.048$

$\Omega_{rad} = 0.00009236$

$\Omega_{\Lambda} = 0.694$

The equations that describe the influence of H, with these conventions and data can be written:

a- Case where we do not separate dark matter from baryons.

$$\frac{1}{H_0}\int_0^{1089} \frac{dz}{\sqrt{0{,}00009236(1+z)^4 \ + 0{,}306(1+z)^3 \ + 0{,}694}}$$

b- Case where we separate dark matter from baryons. The coupling factor is defined by f(z) = [1- tanh(z/1089)]^3

$$\frac{1}{H_0}\int_0^{1089} \frac{dz}{\sqrt{0{,}00009236(1+z)^4 \ + 0{,}258(1+z)^3 \ + 0{,}048(1+z)^3\left(1-\tanh\left(\frac{z}{1089}\right)\right)^3 + 0{,}694}}$$

Value of the integral in case a)

By using the « NIntegrate » command in mathematica, where we set x = z + 1, this can be written:

```
NIntegrate[1 / Sqrt[0.00009236 * x^4 + 0.306 * x^3 + 0.6939], {x, 1, 1090}]
```

The result given by mathematica is: `3.1531142721606287`` → H_0 = 67,27 km/s/Mpc

For the case b), this can be written:

```
NIntegrate[
 1 / Sqrt[0.00009236 * x^4 + 0.258 * x^3 + 0.048 * x^3 * (1 - ArcTan[(x - 1) / 1089]) ^3 +
    0.6939], {x, 1, 1090}]
```

The result given by mathematica is: `3.1690764909827074`` → H_0 = 67,61 km/s/Mpc

The difference is low, (0,5%) but the coupling function, which is arbitrary, aimed just to illustrate the principle of the method.

The result in b) is greater because it is the 1/H function which is evaluated. Indeed, the multiplying factor of H_0 is lower, this implying that H_0 will increase for keeping the product constant. We just modified the function H (z).

Numerical results for D_A and H_0 per these equations

Let us recall the equation ruling the process.

$$D_A = \frac{c}{(1+z)H_0}\int_0^{z*} \frac{dz}{\sqrt{\Omega_{rad}(1+z)^4 \ + \Omega_{dm}(1+z)^3 \ + \Omega_{bar}(1+z)^3 f(z) + \Omega_\Lambda}}$$

In previous calculations, we set c = 1. The dimensional analysis allows to re-introduce c. D_A is a length, z, Ω_i are dimensionless and $1/H_0$ is a time. For homogeneity we multiplied the term on the right, in the above equation, by c (velocity of light 299792458km/s). For the length units, we will select the meter (m) as a common unit and for the time the second (s). Let us write H_0 with these units.[1]

$$H_0 = 6{,}7270\ 10^4\ m/s/(3.0834x10^{22}m) = 2{,}1817x\ 10^{-18}\ s^{-1}. \rightarrow 1\ /H_0 = 0{,}45836\ x\ 10^{18}\ s.$$

The value of the integral is: 3, 1531 and z = 1089, this allows to calculate D_A.

$$D_A = \frac{2{,}99792458x10^8}{(1{,}090)10^3x2{,}1817.10^{-18}}\ 3{,}1531 = 3{,}975x\ 10^{23}\ m = 12{,}89\ Mpc$$

The value of angular size deduced for the first peak of figure 3, is: $\theta = 0{,}0104$ radian $\approx 0{,}6°$. The size D of the sonic horizon is given by = θ x D_A = 0.1341 Mpc

These values are compatible with the well-known values resulting from the Planck mission.

Conclusion

When the current theory (the standard model of cosmology) fails to explain some phenomenon, before discarding it for another, assuming that there is one which would be better (this is appreciated on the set of predictions made by theory), we must ensure that it is used correctly. The case of cosmology is particular, insofar as, for the theory, drastic simplifying assumptions have been made (homogeneity and isotropy on a large scale), essentially for finding analytical solutions! We know that this is highly approximate, even if simulations show that, overall, even “sponge” structures with filaments produce results rather close to what the theory predicts. Nevertheless, caution is in order. In the example given, we see that the impact of the modifications exposed is low on the result. But it is a proposition that serves only as an example. If history has shown that a sole experiment could invalidate a theory, the Michelson-Morley experiment for mechanics which induced special relativity, that of the black body for thermodynamic, which induced quantum mechanics, for general relativity this is not really the case. The tiny anomaly of Mercury's orbit (advance of its perihelion) bothered astronomers a little, but nothing more. A cause, explicable within the framework of classical mechanics was assumed: a flatten shape of the Sun, a planet inside Mercury orbit (Vulcan) that we had not

[1] In a year there is 3600 x 24 x 365,15 ≈ 3.1549 x 10^7 seconds and in a light-year 9.458 x 10^{15} meters. A parsec is 3,26 light-years, i.e., 3.0834x 10^{16} meters, 1 megaparsec is 3.0834x 10^{22} meters.

observed, etc. General relativity arose from Einstein's strong belief that relativity should also apply to gravity! That it fixed the Mercury problem was a "divine surprise" that delighted its author, but the theory was not designed specifically for that. There were other totally unexpected surprises, and that is what made this theory so interesting and powerful!

For the problem of the Hubble constant, if we look at the existing "competing" theories it is not obvious that there is a better one, and some (for quantifying the gravitation) are still under construction, and this, in spite, of considerable research effort devoted to it.

Obviously, the problem is not simple. Of course, a theory is not a truth, it is not definitive, and history has shown how they can be improved, a task that falls to physicists.

This anomaly of the Hubble constant is finally perhaps a chance, because by the nature of the problem which it raises, (laws which seem to depend on z, differently from what one expected), like the examples mentioned previously, it can give us information for a lead towards a new approach.

However, this should not prevent us to verify that the explanation may reside within the framework of the existing theory and that this anomaly would be linked to a poor knowledge of the parameters and that, too, is for the physicist to do.

Appendix : How define a function H (z).

The Friedmann-Lemaître's equation uses the Robertson-Walker's metric, the coordinates of which are (t, r, θ et φ). For substituting z to them (in radial geodesics), let us calculate the Hubble's constant, which in Friedmann-Lemaître formalism is : H = a'(t)/a(t), where a(t) is the expansion of space factor and *a'* its derivative in regard to time *t*, in the Robertson-Walker. metrics, as follows:

$$H = \frac{a'}{a} = \frac{d}{dt}\ln(\frac{a(t)}{a_0}) = \frac{d}{dt}\ln\left(\frac{1}{1+z}\right) = \frac{-1}{1+z}\frac{dz}{dt}$$

With the value of H (for $\Omega_k = 0$):

$$H = H_0\sqrt{(\,\Omega_{rad}(1+z)^4 \;+\Omega_m(1+z)^3 \;+\Omega_\Lambda)}$$

One gets:

$$\frac{dt}{dz} = \frac{-(1+z)^{-1}}{H_0\sqrt{(\,\Omega_{rad}(1+z)^4 \;+\Omega_m(1+z)^3 \;+\Omega_\Lambda)}}$$

This will allow to use z, in place, of t, in many equations.